

Tutorial Assignment 1: Basic Statistical Mechanics

*Handwriting should be legible. Calculations should be explained. Units should be given.
Numerical answers should be given to 4 significant figures.*

1. There are 10 particles in a box. Each particle has two energy levels, ϵ_1 and ϵ_2 . The probability that a particle is at ϵ_1 is 0.6, at ϵ_2 is 0.4.
 - (i) Find the average population of particles at ϵ_1 , and the average population of particles at ϵ_2 . [1]
 - (ii) Suppose that ϵ_1 is 1 J and ϵ_2 is 2 J. Find the average total energy of all particles. [1]

2.
 - (i) How many particles are there in 1 mole of particles? [1]
 - (ii) Find the number of particles in 0.5 mole. [1]
 - (iii) How many moles are in 10^{24} particles? [1]

3. There are 1 mole of particles in a box. Each particle has two energy levels, $\epsilon_1 = 0$ J and $\epsilon_2 = 10^{-23}$ J. The temperature of the box is 1 K.
 - (i) The particles are distributed among the two levels. State the name of this distribution. [1]
 - (ii) Find the Boltzmann factor for each level. [2]
 - (iii) Find the ratio of the populations in the two levels. Give your answer in the form 1:x. [1]
 - (iv) Find the probability that a particle is at ϵ_1 , and the probability that a particle is at ϵ_2 . [2]
 - (v) Find the population of particles at ϵ_1 , and the population of particles at ϵ_2 . [2]
 - (vi) Find the total energy of the particles. [1]
 - (vii) Suppose that half of the particles fall from ϵ_2 to ϵ_1 . Find the new populations. Find the loss in energy of the particles. [3]
 - (viii) This fall from ϵ_2 to ϵ_1 could happen if there is a decrease in temperature. Write down the Boltzmann distribution formula for each level. [1]
 - (ix) Using these formulae and the new populations, solve for the new temperature. [2]

CONSTANTS

Speed of light in vacuum	c	=	$3.00 \times 10^8 \text{ ms}^{-1}$
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7} \text{ Hm}^{-1}$
		=	$4\pi \times 10^{-7} \text{ VsA}^{-1}\text{m}^{-1}$
Permittivity of vacuum	ϵ_0	=	$8.85 \times 10^{-12} \text{ Fm}^{-1}$
		=	$8.85 \times 10^{-12} \text{ AsV}^{-1}\text{m}^{-1}$
Elementary charge	e	=	$1.60 \times 10^{-19} \text{ C}$
Planck constant	h	=	$6.63 \times 10^{-34} \text{ Js}$
	$h/2\pi = \hbar$	=	$1.05 \times 10^{-34} \text{ Js}$
Avogadro constant	N_A	=	$6.02 \times 10^{23} \text{ mol}^{-1}$
Boltzmann constant	k_B	=	$1.38 \times 10^{-23} \text{ JK}^{-1}$
Gas constant	R	=	$8.31 \text{ JK}^{-1}\text{mol}^{-1}$
Unified atomic mass constant	m_u	=	$1.66 \times 10^{-27} \text{ kg}$
		=	931.5 MeVc^{-2}
Electron mass	m_e	=	$9.11 \times 10^{-31} \text{ kg}$
Proton mass	m_p	=	$1.67 \times 10^{-27} \text{ kg}$
Gravitational constant	G	=	$6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$
Acceleration due to gravity	g	=	9.81 ms^{-2}
Bohr magneton	μ_B	=	$9.27 \times 10^{-24} \text{ JT}^{-1}$